Family of Singular Solutions in a SUSY Bulk-Boundary System [a]

SHOICHI ICHINOSE

School of Food and Nutritional Sciences, University of Shizuoka
Yada 52-1, Shizuoka 422-8526, Japan
E-mail: ichinose@u-shizuoka-ken.ac.jp

ABSTRACT

A set of classical solutions of a singular type is found in a 5D SUSY bulk-boundary system. The "parallel" configuration, where the whole components of fields or branes are parallel in the iso-space, naturally appears. It has three *free* parameters related to the *scale freedom* in the choice of the brane-matter sources and the *"free" wave* property of the *extra component* of the bulk-vector field. The solutions describe brane, anti-brane and brane-anti-brane configurations depending on the parameter choice. Some solutions describe the localization behaviour even after the non-compact limit of the extra space. Stableness is assured. Their meaning in the brane world physics is examined.

1 *Mirabelli-Peskin Model* Let us consider the 5 dimensional flat space-time with the signature (-1,1,1,1,1). The space of the fifth component is taken to be S_1, with the periodicity $2l$, and has the Z_2-orbifold condition($x^5 \leftrightarrow -x^5$). We take a 5D bulk theory $\mathcal{L}_{bulk}$ which is coupled with a 4D matter theory $\mathcal{L}_{bnd}$ on a "wall" at $x^5=0$ and with $\mathcal{L}'_{bnd}$ on the other "wall" at $x^5=l$: $S=\int_{-l}^{l}dx^5\int d^4x\{\mathcal{L}_{blk}+\delta(x^5)\mathcal{L}_{bnd}+\delta(x^5-l)\mathcal{L}'_{bnd}\}$.

We take the Mirabelli-Peskin model[2] as an example. The bulk dynamics is given by the 5D super YM theory(a vector field A^M $(M=0,1,2,3,5)$, a scalar field Φ, a doublet of symplectic Majorana fields λ^i $(i=1,2)$, and a triplet of auxiliary scalar fields X^a $(a=1,2,3)$): $\mathcal{L}_{SYM}=-\frac{1}{2}\mathrm{tr}\,{F_{MN}}^2-\mathrm{tr}\,(\nabla_M\Phi)^2-i\mathrm{tr}\,(\bar{\lambda}_i\gamma^M\nabla_M\lambda^i)+\mathrm{tr}\,(X^a)^2+g\,\mathrm{tr}\,(\bar{\lambda}_i[\Phi,\lambda^i])$.

It is known that we can consistently project out $\mathcal{N}=1$ SUSY multiplet by assigning Z_2-parity to all fields in accordance with the 5D SUSY. A consistent choice is given as: $P=+1$ for A^m,λ_L,X^3; $P=-1$ for $A^5,\Phi,\lambda_R,X^1,X^2$ $(m=0,1,2,3)$. Then $(A^m,\lambda_L,X^3-\nabla_5\Phi)$ constitute an $\mathcal{N}=1$ vector multiplet. Especially $\mathcal{D}\equiv X^3-\nabla_5\Phi$ plays the role of *D-field* on the wall. We introduce one 4D chiral multiplet (ϕ,ψ,F) on the $x^5=0$ wall and the other one (ϕ',ψ',F') on the $x^5=l$ wall. These are the simplest matter candidates and were taken in the original paper[2]. Using the $\mathcal{N}=1$ SUSY property of the fields $(A^m,\lambda_L,X^3-\nabla_5\Phi)$, we can find the following bulk-boundary coupling on the $x^5=0$ wall.

$$\mathcal{L}_{bnd}=-\nabla_m\phi^\dagger\nabla^m\phi-\psi^\dagger i\bar{\sigma}^m\nabla_m\psi+F^\dagger F+\sqrt{2}ig(\bar{\psi}\bar{\lambda}_L\phi-\phi^\dagger\lambda_L\psi)+g\phi^\dagger\mathcal{D}\phi+\mathcal{L}_{SupPot}\ ,$$
$$\mathcal{L}_{SupPot}=(\frac{1}{2}m_{\alpha'\beta'}\Theta_{\alpha'}\Theta_{\beta'}+\frac{1}{3!}\lambda_{\alpha'\beta'\gamma'}\Theta_{\alpha'}\Theta_{\beta'}\Theta_{\gamma'})|_{\theta^2}+\mathrm{h.c.}\quad ,\ (1)$$

where $\nabla_m\equiv\partial_m+igA_m,\ \mathcal{D}=X^3-\nabla_5\Phi,\ \Theta=\phi+\sqrt{2}\theta\psi+\theta^2F$. We take the *fundamental* representation for $\Theta=(\phi,\psi,F)$. In the same way, we introduce the coupling between the other matter fields (ϕ',ψ',F') on the $x^5=l$ wall and the bulk fields: $\mathcal{L}'_{bnd}=(\phi\rightarrow\phi',\psi\rightarrow\psi',F\rightarrow F'\ in\ (1))$. We note the interaction between the bulk fields and the boundary ones is *definitely fixed from SUSY.*

[a] Based on the work with AKIHIRO MURAYAMA[1]

2 *Vacuum Structure* Generally the vacuum is determined by the potential part of scalar fields. We first reduce the previous system to the part which involves only scalar fields or the extra component of the bulk vector.

$$\mathcal{L}^{red}_{blk}[\Phi, X^3, A_5] = \mathrm{tr}\left\{ -\partial_M \Phi \partial^M \Phi + X^3 X^3 - \partial_M A_5 \partial^M A_5 + 2g(\partial_5 \Phi \times A_5)\Phi \right. \\ \left. -g^2 (A_5 \times \Phi)(A_5 \times \Phi) - 2\partial_M \bar{c} \cdot \partial^M c - 2ig\partial_5 \bar{c} \cdot [A^5, c] \right\} \quad , \tag{2}$$

where we have dropped terms of X^1 and X^2 because they decouple from other fields. The field c is the ghost field. While $\mathcal{L}_{bnd}$, on the $x^5 = 0$ wall, reduces to

$$\mathcal{L}^{red}_{bnd}[\phi, \phi^\dagger, X^3 - \nabla_5 \Phi] = -\partial_m \phi^\dagger \partial^m \phi + g(X^3_\alpha - \nabla_5 \Phi_\alpha)\phi^\dagger_{\beta'}(T^\alpha)_{\beta'\gamma'}\phi_{\gamma'} + F^\dagger F \\ + \left\{ \frac{m_{\alpha'\beta'}}{2}(\phi_{\alpha'} F_{\beta'} + F_{\alpha'}\phi_{\beta'}) + \frac{\lambda_{\alpha'\beta'\gamma'}}{3!}(\phi_{\alpha'}\phi_{\beta'}F_{\gamma'} + \phi_{\alpha'}F_{\beta'}\phi_{\gamma'} + F_{\alpha'}\phi_{\beta'}\phi_{\gamma'}) + \mathrm{h.c.} \right\} \quad . \tag{3}$$

In the same way, we obtain $\mathcal{L}^{red\,\prime}_{bnd}[\phi', \phi'^\dagger, X^3 - \nabla_5 \Phi]$ on the $x^5 = l$ wall by replacing, in (3), ϕ and $\phi^\dagger$ by ϕ' and $\phi'^\dagger$, respectively.

The vacuum is usually obtained by the *constant* solution of the scalar-part field equation. In higher dimensional models, however, extra-coordinate(s) can be regarded as parameter(s) which should be separately treated from the 4D space-time coordinates. In this standpoint, it is the more general treatment of the vacuum that we allow the x^5*-dependence* on the bulk-part of the solution. We generally call the classical solutions $(\varphi, \chi^3, a_5; \eta, \eta', f, f')$ the *background fields.* [b] They satisfy the *field equations* derived from (2) and (3) (*on-shell* condition). Assuming $\varphi = \varphi(x^5), \chi^3 = \chi^3(x^5), a_5 = a_5(x^5), \eta =$ const$, \eta' =$ const$, f =$ const$, f' =$ const, the field equation of $\mathcal{L}^{red}_{blk} + \delta(x^5)\mathcal{L}^{red}_{bnd} + \delta(x^5 - l)\mathcal{L}^{red\,\prime}_{bnd}$ are given by, for the bulk-fields variation,

$$\delta\Phi_\alpha \quad ; \; -\partial_5 Z_\alpha - g(Z \times a_5)_\alpha = 0 \; , \quad \delta A_{5\alpha} \quad ; \; {\partial_5}^2 a_{5\alpha} - g(\varphi \times Z)_\alpha = 0 \; , \\ \delta X^3_\alpha \quad ; \; \chi^3_\alpha + g(\delta(x^5)\eta^\dagger T^\alpha \eta + \delta(x^5 - l)\eta'^\dagger T^\alpha \eta') = 0 \; , \quad , \tag{4}$$

where $Z_\alpha \equiv -g(\delta(x^5)\eta^\dagger T^\alpha \eta + \delta(x^5 - l)\eta'^\dagger T^\alpha \eta') - \partial_5 \varphi_\alpha + g f_{\alpha\beta\gamma} a_{5\beta}\varphi_\gamma$. The field equations for the boundary-fields part are given by the variations $\delta\phi^\dagger_{\alpha'} \quad (\delta\phi'^\dagger_{\alpha'})$ and $\delta F^\dagger_{\alpha'} \quad (\delta F'^\dagger_{\alpha'})$:

$$d_\beta|_{x^5=0} \times (T^\beta \eta)_{\alpha'} + m_{\alpha'\beta'} f^\dagger_{\beta'} + \frac{1}{2}\lambda_{\alpha'\beta'\gamma'}\eta^\dagger_{\beta'} f^\dagger_{\gamma'} = 0 \; , \; (\eta \to \eta', f \to f' \text{ in the left equation}) \; , \\ f_{\alpha'} + m_{\alpha'\beta'}\eta^\dagger_{\beta'} + \frac{1}{2}\lambda_{\alpha'\beta'\gamma'}\eta^\dagger_{\beta'}\eta^\dagger_{\gamma'} = 0 \; , \; (\eta \to \eta', f \to f' \text{ in the left equation}) \; , \tag{5}$$

where $d_\alpha = (\chi^3 - \partial_5 \varphi + g a_5 \times \varphi)_\alpha$ is the background D-field. From the equation (4), we obtain $\chi^3_\alpha = -g(\delta(x^5)\eta^\dagger T^\alpha \eta + \delta(x^5 - l)\eta'^\dagger T^\alpha \eta')$, and we know $Z_\alpha = d_\alpha$

Before presenting the solution, we note a simple structure involved in them. Under the "*parallel*" circumstance, $a_{5\alpha} \propto \varphi_\alpha \propto \eta^\dagger T^\alpha \eta \propto \eta'^\dagger T^\alpha \eta'$, the equations for $\delta\Phi_\alpha$ and $\delta A_{5\alpha}$ are ${\partial_5}^2 \varphi_\alpha = -g\partial_5(\delta(x^5)\eta^\dagger T^\alpha \eta + \delta(x^5 - l)\eta'^\dagger T^\alpha \eta')$ and ${\partial_5}^2 a_{5\alpha} = 0$. The first one is a static wave equation with "source" fields located at $x^5 = 0$ and l. It is easily integrated

[b] In the background field treatment[3]we expand all fields around the background fields: $\varphi + \Phi, \chi^3 + X^3, a_5 + A_5, \eta + \phi, \eta' + \phi', f + F, f' + F'$

once. $\partial_5\varphi_\alpha = -g(\delta(x^5)\eta^\dagger T^\alpha\eta + \delta(x^5-l)\eta'^\dagger T^\alpha\eta') + \text{const}$. This result was used in ref.[2]. The equation of $a_{5\alpha}$ is a (static) "free" wave equation (no source fields). $a_{5\alpha}$ do *not* receive, in the "parallel" environment, any effect from the boundary sources η, η'. This characteristically shows the difference between the bulk scalar Φ_α and the extra component of the bulk vector $A_{5\alpha}$ in the vacuum configuration.

We first solve (4) with respect to $a_{5\alpha}$ and φ_α. They also give the solutions for χ^3_α and $d_\alpha = Z_\alpha$. Using these results we solve (5) with respect to η, η', f and f' for given values of $m_{\alpha'\beta'}$ and $\lambda_{\alpha'\beta'\gamma'}$. Here we seek a natural solution by requiring that $d_\alpha(=Z_\alpha)$ is *independent of* x^5. Then, from the equation of (4), we have $Z \times a_5 = 0$. It says that we may consider the three cases : 1) $a_{5\alpha}=0$, 2) $Z_\alpha = 0$, 3) $a_{5\alpha} \propto Z_\alpha (\neq 0)$.

The case 3) solution is given by

$$\varphi_\alpha = \bar{\varphi}_\alpha\{c_1[x^5]_p + [x^5-l]_p\} \quad , \quad a_{5\alpha} = \bar{a}_\alpha\{c_2[x^5]_p + [x^5-l]_p\} \quad ,$$
$$\eta = \text{const} \quad , \quad \eta' = \text{const} \quad , \quad \bar{a}_\alpha = c_3\bar{\varphi}_\alpha \quad , \quad \bar{\varphi}_\alpha = \frac{g}{2l}\eta^\dagger T^\alpha\eta = \frac{1}{c_1}\frac{g}{2l}\eta'^\dagger T^\alpha\eta' \quad ,$$
$$\chi^3_\alpha = -g\{\delta(x^5) + c_1\delta(x^5-l)\}\eta^\dagger T^\alpha\eta \ , \ Z_\alpha = d_\alpha = -\frac{g}{2l}(1+c_1)\eta^\dagger T^\alpha\eta \ , \ \partial_5(\delta A_{5\alpha})|_{x^5=0,l} = 0 \quad , \tag{6}$$

where c_1, c_2 and c_3 are three free parameters. The meaning of c_1 is the scale freedom in the "parallel" condition of brane sources $\eta'^\dagger T^\alpha\eta' \propto \eta^\dagger T^\alpha\eta$, and that of c_2 and c_3 is the "free" wave property of $a_{5\alpha}$. The bulk *scalar* configuration influences the boundary source fields through the parameter c_1, whereas the bulk *vector* (5th component) does not have such effect. Instead the latter one satisfies the field equation only within the restricted variation (Neumann boundary condition). This solution includes the case 1) as $c_3 = 0$ and case 2) as $c_1 = -1$. Another special cases are given by fixing two parameters, c_1 and c_2 (keeping the c_3-freedom), as shown in Table 1.

We have solved only (4). When $m_{\alpha'\beta'}$ and $\lambda_{\alpha'\beta'\gamma'}$ are given, the equations (5) should be furthermore solved for η, η', f and f' using the obtained result. The solutions in the second row ($c_1 = -1$) of Table 1 correspond to the SUSY invariant vacuum, irrespective of whether the vacuum expectation values of the brane-matter fields (η and η') vanish or not. For other solutions, however, d_α depends on η or η', hence the SUSY symmetry of the vacuum is determined by the brane-matter fields. The eqs. (5) have a 'trivial' solution $\eta = 0, f = 0$ (or $\eta' = 0, f' = 0$) when $d_\alpha = -\frac{g}{2l}\eta^\dagger T^\alpha\eta$ (or $d_\alpha = -\frac{g}{2l}\eta'^\dagger T^\alpha\eta'$). It corresponds to the SUSY invariant vacuum. If the equations have a solution $\eta \neq 0$ (or $\eta' \neq 0$), it corresponds to a SUSY-breaking vacuum.

We see the bulk scalar Φ is localized on the wall(s) where the source(s) exists, whereas the extra component of the bulk vector A_5 on the wall(s) where the Neumann boundary condition is imposed. The two cases, ($c_1 = -1, c_2 = -1$) and ($c_1 = 1/0, c_2 = 1/0$), are treated in [4].

3 *Stability* In the present approach, ($\mathcal{N} = 1$)SUSY is basically respected. If SUSY is preserved, the solutions obtained previously are expected to be stable, because the force between branes (Casimir force) vanish from the symmetry. In some cases, we can more strongly confirm the stableness from the topology (or index) as follows. We can regard the extra-space size (S^1 radius) l as an *infrared regularization* parameter for the *non-compact* extra-space $\mathbf{R}(-\infty < y < \infty)$. An interesting case is the $l \to \infty$ limit of

$(c_1=-1,c_2=-1)$ in Table 1: $\varphi_\alpha=-\bar{\varphi}_\alpha l\epsilon(x^5)\rightarrow-\frac{g}{2}\eta^\dagger T^\alpha\eta\tilde{\epsilon}(x^5),\ a_{5\alpha}=-\bar{a}_\alpha l\epsilon(x^5)\rightarrow -c_3\frac{g}{2}\eta^\dagger T^\alpha\eta\tilde{\epsilon}(x^5),\ \chi^3_\alpha=-g\delta(x^5)\eta^\dagger T^\alpha\eta,\ Z_\alpha=d_\alpha=0,\ \partial_5(\delta A_{5\alpha})|_{x^5=0}=0$. The above limit is a solution of $S=\int d^4x\int_{-\infty}^{+\infty}dx^5\{\mathcal{L}_{blk}+\tilde{\delta}(x^5)\mathcal{L}_{bnd}\}$, $-\infty<x^5<\infty$, where $\mathcal{L}_{blk}$ and $\mathcal{L}_{bnd}$ are the same as before except that fields are no more periodic. The stableness is clear from the same situation as the kink solution . On the other hand, in the $l\rightarrow\infty$ limit of $(c_1=1/0,c_2=1/0)$ there remains no localization configuration.

4 *Conclusion* In the brane system appearing in string/D-brane theory, the stableness is the most important requirement. We find some stable brane configurations in the SUSY bulk-boundary theory. We systematically solve the singular field equation using a general mathematical result about the free-wave solution in S_1/Z_2-space[1]. The two scalars, the extra-component of the bulk-vector (A_5) and the bulk-scalar(Φ), constitute the solutions. Their different roles are clarified. The importance of the "parallel" configuration is disclosed. The boundary condition (of A_5) and the boundary matter fields are two important elements for making the localized configuration. Among all solutions, the solution $(c_1=-1,c_2=-1)$ is expected to be the thin-wall limit of a kink solution.

In ref.[4,3], the 1-loop effective potential is obtained for the backgrounds $(c_1=-1,c_2=-1)$. In ref.[6], a bulk effect in the 1-loop effective potential is analyzed in relation to the SUSY breaking. We hope the family of present solutions will be used for further understanding of the bulk-boundary system.

	$c_2=-1$ $\partial_5(\delta A_{5\alpha})\|_{x^5=0,l}=0$	$c_2=0$ $\partial_5(\delta A_{5\alpha})\|_{x^5=0}=0$	$c_2=1/0$ $\partial_5(\delta A_{5\alpha})\|_{x^5=l}=0$
$c_1=-1$ $(\eta'^\dagger T^\alpha\eta'$ $=-\eta^\dagger T^\alpha\eta)$	$\partial_5\varphi_\alpha:\ B\bar{B}$ $\partial_5 a_{5\alpha}:\ B\bar{B}$ $d_\alpha=0$	$\partial_5\varphi_\alpha:\ B\bar{B}$ $\partial_5 a_{5\alpha}:\ B$ $d_\alpha=0$	$\partial_5\varphi_\alpha:\ B\bar{B}$ $\partial_5 a_{5\alpha}:\ \bar{B}$ $d_\alpha=0$
$c_1=0$ $(\eta'^\dagger T^\alpha\eta'=0)$	$\partial_5\varphi_\alpha:\ B$ $\partial_5 a_{5\alpha}:\ B\bar{B}$ $d_\alpha=-\frac{g}{2l}\eta^\dagger T^\alpha\eta$	$\partial_5\varphi_\alpha:\ B$ $\partial_5 a_{5\alpha}:\ B$ $d_\alpha=-\frac{g}{2l}\eta^\dagger T^\alpha\eta$	$\partial_5\varphi_\alpha:\ B$ $\partial_5 a_{5\alpha}:\ \bar{B}$ $d_\alpha=-\frac{g}{2l}\eta^\dagger T^\alpha\eta$
$c_1=1/0$ $(\eta^\dagger T^\alpha\eta=0)$	$\partial_5\varphi_\alpha:\ \bar{B}$ $\partial_5 a_{5\alpha}:\ B\bar{B}$ $d_\alpha=-\frac{g}{2l}\eta'^\dagger T^\alpha\eta'$	$\partial_5\varphi_\alpha:\ \bar{B}$ $\partial_5 a_{5\alpha}:\ B$ $d_\alpha=-\frac{g}{2l}\eta'^\dagger T^\alpha\eta'$	$\partial_5\varphi_\alpha:\ \bar{B}$ $\partial_5 a_{5\alpha}:\ \bar{B}$ $d_\alpha=-\frac{g}{2l}\eta'^\dagger T^\alpha\eta'$

Table 1 Various vacuum configurations of the Mirabelli-Peskin model.
$B\bar{B}$, $\bar{B}$ and B correspond to brane-anti-brane, anti-brane and brane respectively.

5 *References*

[1] S. Ichinose and A. Murayama, hep-th/0405065, Phys.Lett.**596B**(2004)123

[2] E.A.Mirabelli and M.E. Peskin, hep-th/9712214, Phys.Rev.**D58**(1998)065002

[3] S. Ichinose and A. Murayama, hep-th/0401011, US-03-08 "Quantum Dynamics of A Bulk-Boundary System"

[4] S. Ichinose and A. Murayama, hep-th/0302029, Phys.Lett.**587B**(2004)121

[5] A. Hebecker, Nucl.Phys.**B632**(2002)101

[6] S. Ichinose and A. Murayama,hep-th/0403080, Phys.Lett.**593B**(2004)242.